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Velocity-dependent restitution coefficient and granular cooling in microgravity

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Abstract – We experimentally investigate the free cooling process occurring in a vibrated granular medium made of inelastic particles in a two-dimensional geometry. Experiments are realized in microgravity to cancel gravitational effects and recorded with the help of a high-speed camera. From the trajectories of the particles, obtained by image analysis, we can determine both the restitution coefficient and the time decay of the energy in the medium as soon as the vibration is cut off. We found evidence at low velocities of a positive slope of the restitution coefficient *vs.* the impact velocity, contrary to the usual approach where it only decreases with the velocity. We also found that the experimental cooling time is also much shorter than the one predicted on the basis of constant or decreasing restitution coefficient. A better agreement between theory and experiment is found if we take into account either the rotational degree of freedom or the velocity dependence of the coefficient of restitution.

Introduction. – A system of inelastic hard spheres is a reference model for the study of granular flows [1,2]. Such a system appears in many physical situations ranging from astrophysics to material processing. One of the interesting phenomena arising in the dynamics of granular matter is the free cooling of a system of inelastic particles which gives rise to clustering instability. This free-cooling process is observed as the evolution of a set of particles, initially in motion, when the external energy supply is removed. The velocities of particles drop to zero due to inelastic collisions which is the starting point of clustering structures reported in numerous works [3,4]. Binary collisions lead the dynamics of the system and, consequently, the energy loss is described by introducing the dissipation during two particles collisions on the basis of an effective restitution coefficient. Despite the difficulties related to varied phenomena existing in this cooling, like clusters, vortices... previous studies have reported the rate of decrease of the energy of the system *vs.* time. The cooling of granular systems *vs.* time is generally presented in three consecutive steps: an initial transient one where the kinetic energy of particles decreases as t^{-2} [5], a second one where vortices develop in the system and the kinetic energy decreases as t^{-1} [6], then a last one where dense clusters appears inducing an inhomogeneous density field in the system [7].

The starting point of these investigations is to relate the rate of decrease of the energy through a rate of collisions n_c with a loss of energy, at each collision, proportional to $(1-e^2)$ where e is the normal restitution coefficient. This rate of collision is given by the Enskog time [8]:

$$t_E = \frac{\sigma\sqrt{\pi}}{2\sqrt{2}\rho V_0\, g(r)}, \tag{1}$$

where σ is the particle diameter, ρ the volume fraction, V_0 the initial average velocity in the medium and $g(r)$ the pair correlation function at contact. The energy decrease *vs.* time can thus be written as $E(\tau)=1/(1+\tau)^2$, where E is the energy normalised by the initial one, $E=T/T_0$, and τ the normalised time: $\tau=(1-e^2)\,t/t_E$ [9]. This last relation assumes a restitution coefficient which is independent of the impact velocity V_i between particles. Note that T is the translational kinetic energy which is used to define the 2D translational granular temperature: $T=1/2mv^2$ and so the decrease of energy is equivalent to a cooling phenomenon.

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Additional studies have also included the rotational part of the energy by introducing a tangential restitution coefficient [10]. But the critical point seems to be the impact velocity dependence of the restitution coefficient, *i.e.* considering the visco-elastic nature of the granular particles. [11,12]. In this situation, the exponent in the energy decrease is reported to be 5/3 instead of 2. The dependence of $e(V_i)$ and its role in the free cooling of a granular medium gave rise to a large number of theoretical and numerical works [13–16]. Some works [17–19] based on theoretical and experimental investigations reported a decrease of e with increasing velocities but others [20,21] found the opposite behavior at small velocities —typically below a few cm/s.

The predicted free cooling of a granular medium were tested against numerical simulations, but to our knowledge, there are no experimental study of this phenomena realized without the drawback of gravity. The aim of this paper is to present a first experimental investigation of this phenomenon realized in microgravity and to interpret the results with the knowledge of all the experimental quantities involved in the theories.

In the first section we describe the experiments performed on a model granular system composed of iron beads in a 2D-configuration. In a second part we compare our results to existing theories and discuss the possible reasons of the observed disagreements between experiments and theory. Finally, we present a set of improvements which are going to be performed in future experimental investigations.

Experiments. – We experimentally investigated the dynamical behavior of a model granular gas submitted to an external periodic vibration [22]. The medium is composed of iron beads with radius $\sigma = 2\,\text{mm}$, enclosed in a 2D cell (fig. 1). The initial volume fraction of the medium is $\rho_i = 19\%$. The cell was chosen with a rounded shape in order to ensure a homogeneous injection of energy in the system in the presence of the vibration. The cell's walls are made of glass in order to cancel electrostatic effects and to minimize the friction between the beads and the walls. Moreover, to remove gravitational effects, the experiments have been performed in zero-gravity situations inside a special equipped airplane undergoing parabolic flights. This experimental situation permits us to avoid density fluctuations like the ones occurring in fluidized beds or strong rolling contributions as the ones encountered in horizontal studies where the particles move over a horizontal vibrated plate. The 2D configuration also forbids the cross-over of particles trajectories.

The cell is submitted to a sinusoidal vibration with different amplitudes and frequencies allowing a range of maximum cell's velocities from 30 cm/s up to 250 cm/s. The motion of granular particles is recorded with the help of a high-speed camera at 470 frames per seconds during about 6 s, giving us a collection of about 3000 pictures per experiment with a picture dimension of 320×320 pixels.

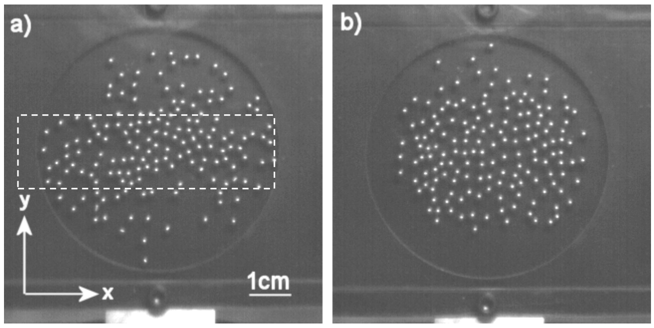

Fig. 1: The 2D cell containing the granular medium is mounted on a vibrating device (along the y-axis) and high-speed video recordings are performed to track the motion of the particles. The whole set-up is located in an airplane undergoing parabolic flights to cancel the effect of gravity (the pictures shown represent typical situations recorded during experiments). a) Vibration is on: the central part of cell contains an almost constant density of particles (dashed region). b) The vibration has been cut off. The motion of particles stops due to inelastic collisions.

Since we are interested in the free cooling occurring in the granular medium, the experiments are performed as followed:

- Prior to microgravity situation, the vibration is switched on (here the particles are mainly located at the bottom of the cell, y being the normal gravity direction).
- Once gravity drops to zero, the particles fill the entire cell and a region of a rather homogeneous density appears in the center of the cell. The video recording is then started.
- Finally, during recording, the external vibration is cut off and we record the slowing-down of the particles.

During the vibration, typical configurations of the experimental cell shows two hot (and dilute) regions at the top and bottom of the cell while a dense region exists in the center of the cell: this experimental configuration gives us the possibility to study a homogeneous bed of particles in contact with two hot regions. Once the vibration is stopped, the particles almost come to rest due to inelastic collisions inducing energy loss. In such cooling granular systems, a well-known effect is the formation of dense clusters of particles. This phenomenon is not clearly observed in our experiments: when the energy input is cancelled we rather observe some alignments of particles along "wavy lines" but not clear regions of high and low density because the main energy loss during collision occurs along the normal direction between two particles. Another reason may come from the rather low initial volume fraction of the particles. Moreover, in experimental situations, some undesired gravity fluctuations still exists giving rise to a collective motion of all particles in a given direction. But a

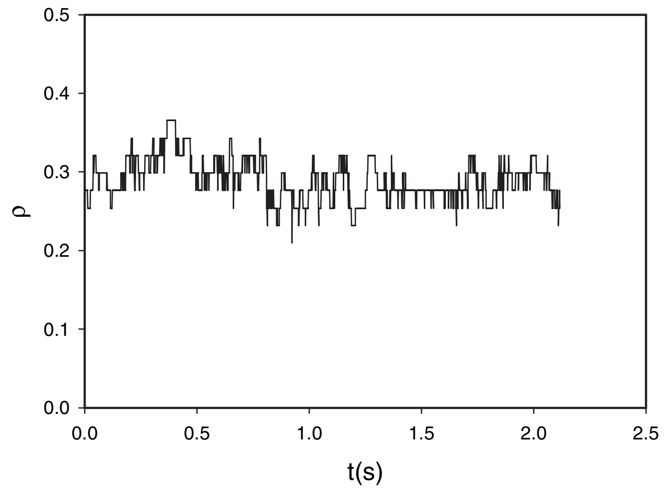

Fig. 2: Time behavior of the volume fraction of particles in the central area of the cell. In this region, we may assume that the volume fraction of the granular medium remains constant.

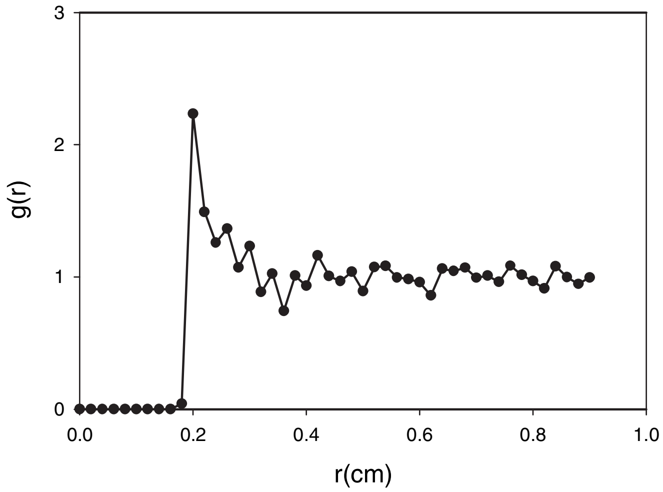

Fig. 3: Experimental pair correlation function $g(r)$ calculated from the positions of the particles. This curve is averaged over all pictures recorded and on the spatial configurations of particles in the central region of the cell. The non-zero value just below the diameter of particles comes from the uncertainty of the determination of positions of the particles.

short time after the vibration has been removed; we generally observe that the particles tend to stop in the center of the cell without evidence for clustering.

The experimental processing is performed by image analysis. Each particle p is individually tracked allowing to obtain the positions $x_p(t)$ and $y_p(t)$ as a function of time. Since the density of particles remains almost constant in the central part of the cell (fig. 2), we will concentrate our investigations only in that region (the dashed white rectangle in fig. 1). It is interesting to note that from these sets of coordinates all experimental parameters considered in theories can be directly determined such as the velocities components, the normal restitution coefficient, e, but also the pair correlation function $g(r)$ for which a typical behavior is shown in fig. 3. Note that the maximum value of $g(r)$ is achieved at the particle diameter which prove that undesired electrostatic effects may be neglected. The small non-zero value of the pair correlation function "before" the particle diameter comes from the uncertainty in the determination of particle's position by image analysis.

Actually in the state of the experiment, only the translational velocities of the particles can be determined. The rotational part is not accessible since we use beads but further experiments, with disks, will lead to this missing part.

The experimental determination of the normal restitution coefficient between particles has been investigated as a function of the relative normal velocities of two colliding particles. We made a statistics on two-particles collisions (with and without the external vibration) by comparing, on experimental trajectories, the directions and the magnitudes of the velocities before and after impact. The behavior of the restitution coefficient *vs.* the normal relative impact velocity is presented in fig. 4. One can note that for high relative

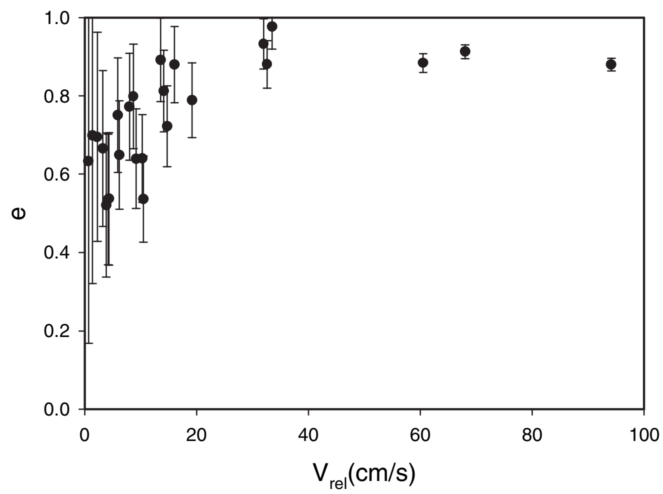

Fig. 4: Experimental dependence of the normal restitution coefficient, e, as a function of the relative normal impact velocity, obtained directly from the trajectories of the particles. A clear decrease of e at low impact velocities is observed.

velocities we recover the restitution coefficient of 0.9 which is the one usually given for stainless steel beads. The most striking feature is that the restitution coefficient sharply decreases when the impact velocity decreases. This situation is well known in the presence of wet particles where $e = 0$ for Stokes number $S_t = (2mV_i)/(3\pi\eta\sigma^2)$ smaller than a critical one, $S_t|_c$ [23] and then rises approximately as $1 - S_t|_c/S_t$. This is explained by the viscous dissipation but, for dry particles, most experiments report a decrease of the restitution coefficient when increasing the impact velocity. Actually most of the experiments are made in the presence of

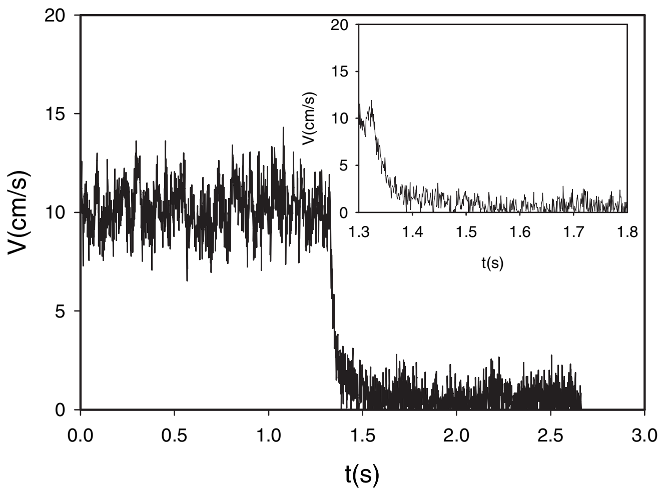

Fig. 5: Time dependence of the average magnitude of the translational velocity of particles calculated from the positions in the central part of the cell. The vibration is cut off during recording and the decay of the energy in the granular system is clearly observed (max cell velocity of 74.6 cm/s). The insert represents a zoom of the curve when the external vibration is stopped.

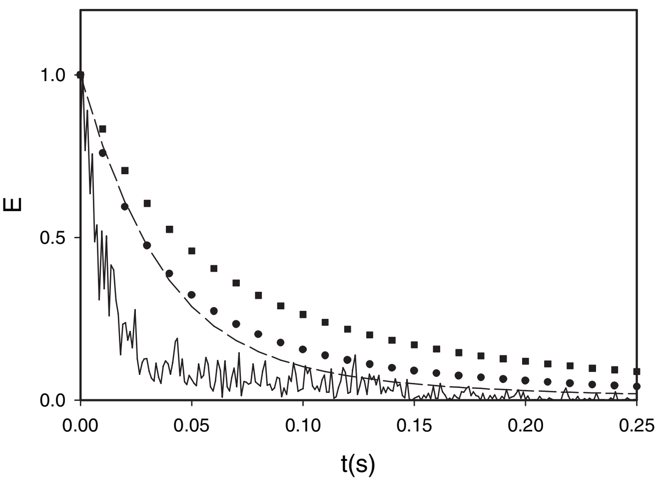

Fig. 6: Direct comparisons between experiments (plain curve) and theory on the energy decay in the granular medium. Black squares: theoretical predictions on the basis of a constant restitution coefficient. Black circles: theoretical predictions including the rotational kinetic energy of the particles. Dashed line: theory considering only the translational part of the energy but including a velocity-dependent restitution coefficient with a similar behavior to the one presented in fig. 4.

gravity with impact velocities larger than 1 m/s (for a height $h = 5$ cm the impact velocity of a bead on the plane: $V_i = \sqrt{2gh}$ is already 1 m/s). In a recent paper [21] on the restitution coefficient between two dry beads attached by strands in a pendulum device, the authors also found a lower restitution coefficient at low velocities (typically below 20 cm/s). They interpret this behavior by van der Waals adhesion between the flattened parts of the surface roughness. Our results, obtained in microgravity without experimental drawbacks, well confirm these findings.

Finally, in fig. 5 the average velocity of the particles in the central part of the cell before and after stopping the vibration is presented. One can observe the rapid decay of the average velocity. The non-zero value measured for "long times" comes from the small gravity fluctuations occurring during the parabolic flight.

Comparison with theory. – In a first step, we consider the energy decay assuming a constant restitution coefficient (typically $e = 0.9$ for stainless steel beads). The time dependence of the energy is predicted to behave as $E(\tau) = 1/(1+\tau)^2$, where τ includes the Enskog time. With our experimental set-up, we can access all parameters involved in this theoretical description. A quantitative comparison with experiments is presented in fig. 6 (squared symbols) for a cell velocity of 75 cm/s and with the following experimental values: $\rho = 0.297 \pm 0.027$, $V_0 = (0.11 \pm 0.01)$ m/s and $g(r) = 2.23 \pm 0.02$.

We see in the fig. 6 that the decrease of energy observed experimentally is much faster than the one predicted with a constant restitution coefficient. The first possible reason for these differences could come from the friction of the particles on the glass walls of the cell (*i.e.* an additional loss of energy not due to the inelastic collisions). Nevertheless, a precise analysis of the trajectory of single particles after the vibration has been cut off shows a linear motion at constant speed in between two collisions of particles. It is illustrated in fig. 7 where we have plotted the (x, y) positions of one particle over a time interval of the same order has the characteristic decay time of the energy (about 0.15 s). The change in velocity between the two extremities of the curve is below the uncertainty given by the determination of the positions,

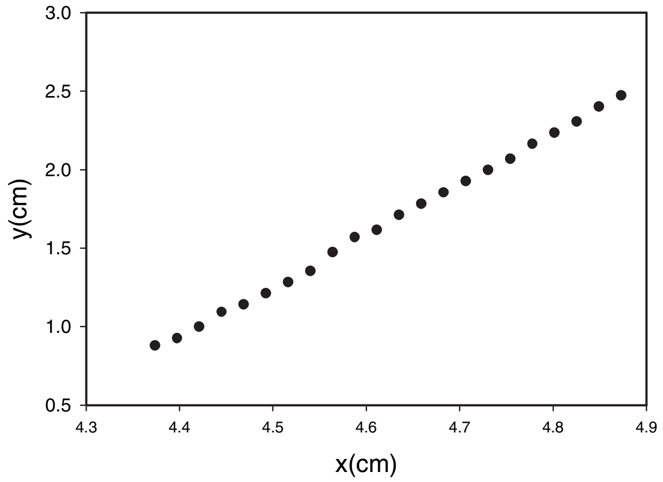

Fig. 7: Plot of the trajectory of a single particle during cooling retrieved from recorded experimental pictures. No collision occurs on the whole trajectory and the time interval between each position is 0.009 s. The velocity of the particle remains almost constant during the whole motion.

and we have checked that point on several trajectories, so we exclude that friction of the beads on the walls could play a significant role on the cooling process. The Enskog collision time can be calculated from eq. (1). One gets a value of $t_E = 0.0172\,\text{s}$. In order to check this value, we have performed a large statistics on our experiments to find the average time interval separating two consecutive collisions by tracking different particles in the central part of the cell. We found an average time interval of $(0.0127 \pm 0.0021)\,\text{s}$ by direct measurements in rather good agreement with the theoretical value.

A possible reason for this discrepancy could be the presence of the rotational kinetic energy which also dissipates a part of the energy through the surface roughness of the particles [9]. A tangential restitution coefficient, β, is then introduced to characterise the effect of the rotation of the particles during a collision. In this case, the time-dependence of the energy (translational and rotational) is determined from a system of coupled differential equations (eq. (15) in ref. [9]) where all parameters can be obtained experimentally, except β. We have solved numerically this system of equations introducing our experimental values with $\beta = 0.1$ (fig. 6, black circles). Note that if we cancel the rotation (*i.e.* $\beta = -1$), the solution is exactly the same as the one predicted with a constant normal restitution coefficient e. We can notice that the energy decreases more rapidly but the effect of the rotational kinetic energy is quite small —we have checked that it remains small whatever the value of β— and it is still not sufficient to fit the experimental behavior.

Finally, in order to consider the velocity dependence of the restitution coefficient determined experimentally from our experiments, we can express the rate of decrease of the translational kinetic energy T as

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -n_c \left(1 - e^2\right) T \tag{2}$$

with n_c the collision rate of binary collisions. In 2D, we have $n_c = (4V\rho g(r))/(\pi\sigma)$ with V the average velocity. Moreover, introducing the normalised energy $E = T/T_0$ and the velocity ratio $V/V_0 = \sqrt{T/T_0}$, the rate of decrease of energy can be rewritten in the form

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{2g(r)\rho V_0}{\sigma}\sqrt{\frac{2}{\pi}}\left(1 - e\,(E)^2\right)E^{3/2}, \tag{3}$$

where now e is assumed to depend on the normal relative velocity. We shall assume that the average relative velocity has the same order of magnitude as the average velocity; then from the curve in fig. 4, a fit of the restitution coefficient *vs.* the normalized energy gives $e(E) = 0.82 - 0.5e^{-2.5E}$. Introducing this last relation in eq. (3) and solving it numerically gives the behavior presented in fig. 6 (dashed line). Compared to the case including the rotation, we observe a more pronounced decrease of the energy with time. This is understandable since we observed that during cooling the restitution coefficient decreases, amplifying the loss of energy. Of course eq. (3) is obtained from a crude approximation based on average velocity instead of the probability distribution of velocities, furthermore it is also likely that the tangential restitution coefficient will also depend on the relative angular velocities of the colliding particles. A complete theory should include both rotational and translational degrees of freedom with the correct velocity dependence of restitution coefficients. In a ongoing work we are going to measure the rotational energy and the velocity dependence of the tangential coefficient of restitution.

Conclusion. – We have reported an experimental investigation of the granular cooling in a 2D model granular medium made of iron beads. Experiments have been performed in zero-gravity situations. The cell is initially submitted to an external vibration which is cut off during high-speed video recording. Determining the trajectory of each particles by fast video camera recording allowed to get a direct measurement of the energy decay in the medium due to inelastic collisions and also to measure the dependence of the restitution coefficient on the relative velocity. We have found that, at low velocities, the restitution coefficient increased with the velocity before reaching a plateau. Experimental curves of the energy decay have been compared to the predictions of existing theoretical model including, or not, the rotation of particles. The experimental relaxation time of the energy is smaller than the theoretical one. On the other hand, including either the coupling between rotation and translation or the velocity dependence on the restitution coefficient significantly improve the agreement with the experiment. Nevertheless a more complete theory is needed to explain the experimental behavior. Additional experimental investigations in microgravity have also to be performed; in order to get information on the decrease of the rotational energy and the measurement of the velocity dependence of the restitution coefficient. An ongoing work on similar experiments with disks (instead of beads) carrying an optical marker will allow us to determine all the needed quantities.

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